Lecture 10: Application Case Study: Deep Learning
Objective

- to learn the implementation of a convolutional neural network
  - basic structure and levels of parallelism
  - inference
  - training
  - optimizations

(forward direction is review for those who took 408)
Perspective is Important

Chips are cheaper than ever.

Unlike humans, digital systems offer

• high-speed computation,
• low capital investment
  (purchase vs. training a human), and
• negligible operations cost (no salary!).

If computer outperforms (or even matches) a human, use a computer.

Industry has done so for 40-50 years now.
What is Machine Learning?

**machine learning**: important method of building applications whose logic is not fully understood

**Usually** for problems for which **incorrect output** does **not** have **catastrophic** consequences.

Typically by **example**:
- **use labeled data** (matched input-output pairs)
- **to represent** desired **relationship**.
Machine Learning Separates into Two Phases

Two phases:

• **training** / learning phase:
  iteratively adjust program logic to produce desired/approximate answers (called **training**)

• **inference** / deployment phase:
  apply learned function to solve new problems (for new input data)
Types of Learning Tasks

• classification
  – Map each input to a category
  – Ex: object recognition, chip defect detection

• regression
  – Numerical prediction from a sequence
  – Ex: predict tomorrow’s temperature

• transcription
  – Unstructured data into textual form
  – Ex: optical character recognition
More Advanced Learning Tasks

• translation
  – Convert a sequence of symbols in one language to a sequence of symbols in another

• structured output
  – Convert an input to a vector with important relationships between elements
  – Ex: natural language sentence into grammatical structure

• others
  – Anomaly detection, synthesis, sampling, imputation, denoising, density estimation, genetic variant calling
Test Cycle Time is Important

You’ve all written code…
• code, test, code, test, code, test
• integrate, test, test, test
• and test again!

But how long is the code, test cycle? Depends what you’re building.

What’s your longest?
Your Cycle Times are Probably Small

In college, 10k lines took ½ hour to compile on my PC. In grad. school, 100k lines took
• ½ hour to compile on my workstation, or
• 2 minutes on our cluster (research platform).
In ECE435 (networking lab), students needed
• ½ hour to reinstall Linux after a bad bug.
• (Ever had a good bug?)
Gene sequencing / applications can take two weeks.

We’re all a little spoiled…
Why Machine Learning Again?

In 2007, **programmable GPUs** accelerated the training cycle.

- Today, new chip designs for learning applications have further accelerated.
- Led to a resurgence of interest
- in Computer Vision, Speech Recognition, Document Translation, Self Driving Cars, Data Science…
- all tasks that human brains solve regularly, but for which we have struggled to express solutions systematically.
Many Problems are Still Hard

**Speed is not a panacea.**

- Many **tasks still require human insight**
  - for network structure and feature selection
  - for effective input and output formats, and
  - for production of high-quality labeled data.
- Other trends sometimes help: ubiquitous computing enables crowdsourcing, for example.
Many Problems Have Systematic Solutions

Example: building a Boolean function from a truth table.

<table>
<thead>
<tr>
<th>Input</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
What if We Lack a Truth Table?

- Make enough observations to construct a rule
  - 000 → 0
  - 011 → 0
  - 100 → 1
  - 110 → 0

- If we cover all input patterns, we can construct a truth table!
Some Problems Too Large to Sample

LeNet-5, a convolutional neural network for hand-written digit recognition

One input is $32 \times 32 \times 8$ bits. $1 \text{ billion } (2^{30}) \text{ images covers...}$

Truth table requires $2^{8192}$ rows! $0\% \text{ of the input space.}$
Labeled Data Provide a Partial Specification

LeNet-5
- tries to look for features that indicate
- which digit is represented by an input image.

**Labeled data give a partial specification** …
- somewhat like random testing, but
- we need examples and
- counterexamples with “correct” labels.
Inputs and Labels May be Biased

LeNet-5*, for example, used

• US Census Bureau and

• high school student samples.

Does that include all versions shown above?

(Actually it does, but you get the point.)

*See yann.lecun.com/exdb/mnist, or the original paper:
Who Defines Correctness of Labeled Data?

Who provides the labels?

Grad students!

16-wk intern. × 120 hrs/wk × 3600 s/hr × 0.1 labels/s
= 691,200 labeled inputs = 1 paper!

Or maybe undergraduates.

These days … crowdsourcing?
Output is a Probability for Each Category

For problems such as **classification**
- (LeNet-5: classify image as digit from 0 to 9)
- **networks** typically **produce a vector of probabilities** (one for each category).

Labels are usually 100% of some category, but not always. Let’s vote on this one… (crowdsourcing may not produce binary answers).
Difficulty Often Depends on Context

Context is also important:
• if you look carefully (at, say, the LeNet-5 paper),
• you’ll see that one also has to capture ‘non-digits.’

Labeled examples of non-digits
• may also help improve training
• if the inference use may include input
• other than the expected categories.

Analogy: it’s easier to grade multiple choice than to grade numerical answers than to grade essay questions, as the space of possible answers grows.
Focus on Neural Networks

**Machine learning** is a huge area
- with **many techniques and approaches**.
- We **focus on neural networks**
- mostly **because GPUs** have come to **dominate** the computation of **these systems**.

Both training and inference using neural networks
- requires **fairly regular computation** to which
- we can apply what we have learned so far.
New Opportunities Have Arisen from Use of GPUs

Deep Neural Networks (DNNs) executing on GPUs
• have also created a variety of opportunities,
• including simultaneous inference and training,
• as when they are dynamically tuned
  – to a particular phone owner’s voice and speech patterns
  – while simultaneously providing speech recognition services.
Neural Networks Model Human Neural Activity

What’s a “neural” network?
An interconnection—usually in layers—of many individual linear classifiers modeled on human neurons.

For training, we want continuous, piecewise differentiable functions with non-zero derivatives, so the neural “activation function” (sign in the next slide) must be chosen carefully.
Perceptron is a Simple Example

Example: a perceptron

\[ y = \text{sign}(W \cdot x + b) \quad \theta = \{W, b\} \]

Dot product: \( y = \text{sign}(W \cdot x) \)
Scalar addition: \( + b \)

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One Choice: Sigmoid/Logistic Function

Until about 2017,

- **sigmoid / logistic function** most popular

\[ f(x) = \frac{1}{1+e^{-x}} \quad (f: \mathbb{R} \rightarrow (0,1)) \]

for replacing the sign function.

- Once we have \( f(x) \), finding \( df/dx \) is easy:

\[
\frac{df(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = f(x) \frac{e^{-x}}{1 + e^{-x}} = f(x)(1 - f(x))
\]
Today’s Choice: ReLU

In 2017, most common choice became
• **rectified linear unit / ReLU / ramp function**
  \[ f(x) = \max(0, x) \quad (f: \mathbb{R} \rightarrow \mathbb{R}^+) \]
  which is much faster (no exponent required).
• A smooth approximation is **softplus/SmoothReLU**
  \[ f(x) = \ln(1 + e^x) \quad (f: \mathbb{R} \rightarrow \mathbb{R}^+) \]
  which is the integral of the logistic function.
• Lots of variations exist. See Wikipedia for an overview and discussion of tradeoffs.
Use Softmax to Produce Probabilities

How can sigmoid / ReLU produce probabilities?

They can’t.

• Instead, given output vector $Z = (z[0], \ldots, z[C-1])^*$,
• we produce a second vector $K = (k[0], \ldots, k[C-1])$
• using the softmax function

$$k[i] = \frac{e^{z[i]}}{\sum_{j=0}^{C-1} e^{z[j]}}$$

Notice that the $k[i]$ sum to 1.

*Remember that we classify into one of C categories.
Softmax Derivatives Needed to Train

We also need the derivatives of softmax,

\[
\frac{dk[i]}{dz[m]} = k[i](\delta_{i,m} - k[m]),
\]

where \( \delta_{i,m} \) is the Kronecker delta (1 if \( i = m \), and 0 otherwise).
Fully Connected Layers Not Always Useful

Consider a $250 \times 250$ image treated as 1D vector.

- **Fully connected** layer is huge:
  - $62,500 \ (250^2)$ weights per node!
  - Comparable number of nodes gives $\sim 4B$ weights total!

- Need $>1$ hidden layer? Bigger images?

  **Too much computation, and too much memory.**
Apply Idea of Convolution to Neural Networks

Feature detection in image processing uses convolution.

Can we do the same in neural networks?
Differences Between CNN and Traditional Network

Convolutional neural networks (CNNs) offer three key differences from traditional layers:

- **spatial relationship between inputs and outputs** (usually squashed into 1D),
- **any output affected only by spatially local inputs** (within mask size), and
- **weights shared across all outputs** (common mask weights).
Convolution Naturally Supports Varying Input Sizes

**Traditional** perceptron **layers**
- have fixed structure, so
- number of inputs / outputs is fixed.

Convolution enables variably-sized inputs
(observations of the same kind of thing)
- audio recordings of different lengths,
- images with more/fewer pixels, and so forth.

**We focus on CNN layers in Lab 7.**
# Example Convolution Inputs

<table>
<thead>
<tr>
<th></th>
<th>Single-channel</th>
<th>Multi-channel</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1D</strong></td>
<td>audio waveform</td>
<td>Skeleton animation data:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1-D joint angles for each joint</td>
</tr>
<tr>
<td><strong>2D</strong></td>
<td>Fourier-transformed audio data</td>
<td>Color image data:</td>
</tr>
<tr>
<td></td>
<td>Convolve over frequency axis:</td>
<td>2D data for R,G,B channels</td>
</tr>
<tr>
<td></td>
<td>invariant to frequency shifts</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Convolve over time axis:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>invariant to shifts in time</td>
<td></td>
</tr>
<tr>
<td><strong>3D</strong></td>
<td>Volumetric data</td>
<td>Color video: 2D data across 1D time</td>
</tr>
<tr>
<td></td>
<td>(example: medical imaging)</td>
<td>for R,G,B channels</td>
</tr>
</tbody>
</table>

Deeplearningbook.org, ch 9, p 355
Convolution Layers in LeNet-5

LeNet-5 has three convolution layers (C1, C3, C5) used to detect features (all) and break symmetry (C3).
Subsampling Layers in LeNet-5

The subsampling layers (S2, S4) help with scale invariance and reduce the amount of data without losing much information.
Fully Connected Layer in LeNet-5

The fully connected layer (F6) is the “hidden” layer in a traditional neural network—implements the unknown function from features to digits.
OUTPUT Layer in LeNet-5

The OUTPUT layer is (F6) uses “Radial Basis Functions” to push digit identification back into the network.
Brains Evolve Architecturally to Learn Faster

**Neural network training is like training and evolving brains.**

Over time, our brains have evolved
- to be good at learning things that are important,
- such as human language and physics.

One can actually identify connections across human languages to physical concepts, such as the differences in behavior between solids and liquids, spatial reasoning, and other important features of our universe.
Neural Networks Train to Produce Output

In neural networks,

• creating a desired output layer
• that represents information (as labels)
• then allows training to produce that information.

(Which then sometimes tells us about the network architecture needed for success, but we won’t look at that.)
Forward Propagation in a Convolution Layer

Weights $W$
- $M$ feature maps
- $C$ channels per map
- $K \times K$ pixels per channel

Input $X$
- $B$ images
- $C$ channels per image
- $H \times W$ pixels per channel

Convolutional Layer

Convolve $W$ with $X$ and sum over channels

Output Size
- $H_{out} = H - K + 1$
- $W_{out} = W - K + 1$

Convolution Output $Y$
- $B$ images
- $M$ features per image
- $H_{out} \times W_{out}$ values per feature

Input-output feature map connections may or may not be complete.
Only Compute with Full Mask/Kernel

Compute only this part of Y.

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Inputs Inflated to Provide Padding

In practice, inputs are sometimes inflated to prepare for this type of convolution!

LeNet-5 input

• starts as $20 \times 20$ monochrome, which are
• scaled to $28 \times 28$, which introduces antialiasing and thus 8-bit grayscale, and then
• padded to $32 \times 32$ to ensure that convolution filters can center on stroke data.
Example of the Forward Path of a Convolution Layer

Output Size
\[ H_{out} = H - K + 1 \]
\[ = 3 - 2 + 1 = 2 \]
\[ W_{out} = W - K + 1 \]
\[ = 3 - 2 + 1 = 2 \]

Convolution Output Y
\[ B=1 \text{ image} \]
\[ M=2 \text{ features per image} \]
\[ H_{out} \times W_{out} = 2 \times 2 \text{ values per feature} \]

Weights W
\[ M=2 \text{ feature maps} \]
\[ C=3 \text{ channels per map} \]
\[ K \times K = 2 \times 2 \text{ pixels per channel} \]

Input X
\[ B=1 \text{ image} \]
\[ C=3 \text{ channels} \]
\[ H \times W = 3 \times 3 \text{ pixels per channel} \]
Sequential Code: Forward Convolutional Layer

```c
void convLayer_fwd(int B, int M, int C, int H, int W, int K, float* X, float* W, float* Y) {
    int H_out = H - K + 1; // calculate H_out, W_out
    int W_out = W - K + 1;

    for (int b = 0; b < B; ++b) // for each image
        for(int m = 0; m < M; m++) // for each output feature map
            for(int h = 0; h < H_out; h++) // for each output value (two loops)
                for(int w = 0; w < W_out; w++) {
                    Y[b, m, h, w] = 0.0f; // initialize sum to 0
                    for(int c = 0; c < C; c++) // sum over all input channels
                        for(int p = 0; p < K; p++) // KxK filter
                            for(int q = 0; q < K; q++)
                                Y[b, m, h, w] += X[b, c, h + p, w + q] * W[m, c, p, q];
                }
}
```

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A Small Convolution Layer Example

Image \( b \) in mini batch

\[ X[b,0,_,_] \]

\[ X[b,1,_,_] \]

\[ X[b,2,_,_] \]

\[ W[0,0,_,_] \]

\[ W[0,1,_,_] \]

\[ W[0,2,_,_] \]

\[ Y[b,0,_,_] \]

\[ Y[b,1,_,_] \]

\[ Y[b,2,_,_] \]

output map

input channel

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A Small Convolution Layer Example $c = 0$

\[\begin{array}{cccc}
1 & 2 & 0 & 1 \\
1 & 1 & 3 & 2 \\
0 & 2 & 2 & 0 \\
2 & 1 & 0 & 3 \\
\end{array} \]

\[\begin{array}{cccc}
1 & 1 & 1 \\
2 & 2 & 3 \\
2 & 1 & 0 \\
\end{array} \]

\[\begin{array}{cccc}
1 & 2 & 1 & 0 \\
0 & 2 & 1 & 0 \\
1 & 1 & 0 & 2 \\
2 & 1 & 0 & 3 \\
\end{array} \]

\[\begin{array}{cccc}
1 & 2 & 3 \\
1 & 1 & 0 \\
3 & 0 & 1 \\
\end{array} \]

\[\begin{array}{cccc}
1 & 2 & 1 & 0 \\
0 & 1 & 3 & 2 \\
3 & 3 & 2 & 0 \\
1 & 3 & 2 & 0 \\
\end{array} \]

\[\begin{array}{cccc}
0 & 1 & 1 \\
1 & 0 & 2 \\
1 & 2 & 1 \\
\end{array} \]

\[\begin{array}{cccc}
1 & 1 & 1 \\
2 & 2 & 3 \\
2 & 1 & 0 \\
\end{array} \]

\[\begin{array}{cccc}
3+13+2 \\
18 \\
? \\
? \\
? \\
\end{array} \]
A Small Convolution Layer Example \( c = 1 \)
A Small Convolution Layer Example $c = 2$
Parallelism in a Convolution Layer

Output feature maps can be calculated in parallel, but usually a small number—not sufficient to utilize GPU. All output feature map pixels can be calculated in parallel:
• all rows in parallel, and
• all pixels in each row in parallel;
• a large number, but diminishes in deeper layers.
All input feature maps can be processed in parallel, but need atomic operations or tree reduction.

Different layers may demand different strategies.
Design of a Basic Kernel

- Each block computes
  - a tile of output pixels for one feature
  - \texttt{TILE\_WIDTH} pixels in each dimension.
- Grid’s X dimension maps to \( M \) output feature maps.
- Grid’s Y dimension maps to the tiles in the output feature maps (linearized order).
- (Grid’s Z dimension is used for images in batch, which we omit from slides.)
Example of Block Y Dimension Mapping

Assume

• \( M = 4 \) (4 output feature maps),
• thus 4 blocks in the X dimension, and
• \( W_{out} = H_{out} = 8 \) (8×8 output features).

If \( TILE\_WIDTH = 4 \),
we also need 4 blocks in the Y dimension:
• for each output feature,
• top two blocks in each column calculates the top row of tiles, and
• bottom two calculate the bottom row.
Host Code for a Basic Kernel: CUDA Grid

Consider an output feature map:

- width is \texttt{W\_out}, and
- height is \texttt{H\_out}.
- Assume these are multiples of \texttt{TILE\_WIDTH}.

Let \texttt{X\_grid} be the number of blocks needed in X dim (5 above).
Let \texttt{Y\_grid} be the number of blocks needed in Y dim (4 above).
Host Code for a Basic Kernel: CUDA Grid

(Assuming \(W_{\text{out}}\) and \(H_{\text{out}}\) are multiples of \(TILE\_WIDTH\).)

// 4 for small examples.
#define TILE_WIDTH 16
// horiz. and vert. tiles per output map
W_grid = \(W_{\text{out}}/TILE\_WIDTH\);
H_grid = \(H_{\text{out}}/TILE\_WIDTH\);
Y = H_grid * W_grid;
// output tile for untiled code
dim3 blockDim(TILE_WIDTH, TILE_WIDTH, 1);
dim3 gridDim(M, Y, 1);
ConvLayerFwd_Kernel<<<gridDim,blockDim>>>(...);
Partial Kernel Code for a Convolution Layer

```c
__global__ void ConvLayerFwd_Basic_Kernel
    (int C, int W_grid, int K, float* X, float* W, float* Y)
{
    int m = blockIdx.x;
    int h = (blockIdx.y / W_grid) * TILE_WIDTH + threadIdx.y;
    int w = (blockIdx.y % W_grid) * TILE_WIDTH + threadIdx.x;
    float acc = 0.0f;
    for (int c = 0;  c < C; c++) {  // sum over all input channels
        for (int p = 0; p < K; p++)  // loop over KxK filter
            for (int q = 0; q < K; q++)
                acc += X[c, h + p, w + q] * W[m, c, p, q];
    }
    Y[m, h, w] = acc;
}
```

Some Observations

**Enough parallelism**
- if the total number of pixels
- across all output feature maps is large
- (often the case for CNN layers).

Each input tile
- loaded M times (number of output features), so
- **not efficient in global memory bandwidth,**
- but block scheduling in X dimension
  should give cache benefits.
Strategies for Lab 7

1. Use tiled 2D convolution with masks in constant memory.
2. Switch to matrix multiply (explanation follows):
   - “unroll” input to enable use of matrix multiplication and
   - use shared memory tiling to perform multiplication.
3. Unroll during load:
   - integrate “unrolling” into loading a matrix tile
   - (unrolled matrix is only conceptual).
4. Use joint register-shared memory tiling.
   Code must work for any credit,
   but (only) last step needed for full credit.
Thread Computes One Pixel of One Output Feature

**Input X**

- B images
- C channels per image
- H×W pixels per channel

**Weights W**

- M feature maps
- C channels per map
- K×K pixels per channel

**Convolutional Layer**

**Convolve W with X and sum over channels**

**Output Y**

- B images
- M features per image
- \( H_{out} \times W_{out} \) values per feature

- \( H_{out} = H - K + 1 \)
- \( W_{out} = W - K + 1 \)

**Consider one image b.**

One thread computes one value of Y for one feature.
Reorganize Input X for Matrix Multiplication

**Input X**
- B images
- C channels per image
- H×W pixels per channel

**Weights W**
- M feature maps
- C channels per map
- K×K pixels per channel

**Multiply W by unrolled X**

**Unrolled Input X**
- B images
- H_{out}×W_{out} pixels per image
- C×K×K values per pixel

Each thread needs a K×K block of X for each input channel.

Note: Likely to be larger than X, since values are replicated.
Matrix Multiplication for a Convolution Layer

\[
\begin{array}{ccc}
1 & 1 & 2 \\
1 & 1 & 2 \\
0 & 1 & 0 \\
1 & 0 & 1 \\
0 & 2 & 1 \\
0 & 2 & 1 \\
1 & 1 & 0 \\
1 & 2 & 0 \\
\end{array}
\quad \begin{array}{ccc}
14 & 20 & 24 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
\end{array}
\quad \begin{array}{ccc}
1 & 2 & 0 \\
1 & 2 & 0 \\
1 & 2 & 0 \\
1 & 2 & 0 \\
1 & 2 & 0 \\
1 & 2 & 0 \\
1 & 2 & 0 \\
1 & 2 & 0 \\
\end{array}
\]

\[
\begin{array}{ccc}
11 & 12 & 11 \\
10 & 01 & 1 \\
12 & 12 & 1 \\
10 & 01 & 1 \\
12 & 12 & 1 \\
10 & 01 & 1 \\
12 & 12 & 1 \\
10 & 01 & 1 \\
\end{array}
\quad \begin{array}{ccc}
1 & 1 & 0 \\
1 & 1 & 0 \\
1 & 1 & 0 \\
1 & 1 & 0 \\
1 & 1 & 0 \\
1 & 1 & 0 \\
1 & 1 & 0 \\
1 & 1 & 0 \\
\end{array}
\quad \begin{array}{ccc}
14 & 20 & 24 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
12 & 24 & 17 \\
\end{array}
\]

\[X \otimes W = Y\]
Simple Matrix Multiplication

Each product matrix element is an output feature map pixel.

This inner product generates element 0 of output feature map 0.
Tiled Matrix Multiplication $2\times2$ example

Each block calculates one output tile – 2 elements from each output map,

Each input element reused 2 times.

Each filter weight reused 2 times.
Tiled Matrix Multiplication 2×4 example

Each block calculates one output tile – 4 elements from each output map.

Each input element reused 2 times.

Each filter weight reused 4 times.

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Unrolling May Boost Your Performance

As we will show, the advantages of “unrolling” are

• to leverage reuse of input values across output features

• without the complexity of 3D convolution, and

• to fit joint tiling into a known pattern.

(You may be able to do as well or better by solving the general convolution, but previous student teams have not succeeded.)
Analyzing the Amount of Input Replication

An output feature map has $H_{out}W_{out}$ elements, and
- each element requires a copy of $K^2$ input elements.
- Each copy is used across all $M$ output feature maps.

The number of unrolled input elements is then
- $H_{out}W_{out}K^2$ for each of the $C$ input feature maps.
- The original input feature maps contain a total of $C(H_{out} + K - 1)(W_{out} + K - 1)$ elements, so

$$H_{out}W_{out}K^2 / [(H_{out}+K–1)(W_{out}+K–1)] \times \text{replication}.$$
Analysis of Small Example

For our toy example,

- $H_{\text{out}} = 2$, $W_{\text{out}} = 2$, and $K = 2$.
- $C=3$ input features.
- “Unrolled” input contains $3 \times 2^4 = 48$ elements.
- Replication factor is $48/3^3 = 1.78\times$. 

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LeNet-5 C1 Layer Unrolling Replication: 19.1×

LeNet-5 C1

- 32×32 input, C=1 feature
- 5×5 filters (K=5)
- 28×28 outputs, M=6 features

Filter matrix: \( M \times CK^2 = 6 \times 25 \)

Unrolled input matrix: \( CK^2 \times H_{out} W_{out} = 25 \times 784 \)

Input replication: \( 25 \times 784/1024 = 19.1 \times \)
LeNet-5 C3 Layer Unrolling Replication: 12.8×

LeNet-5 C3

- 14×14 input, C=6 features
- 5×5 filters (K=5)
- 10×10 outputs, M=16 features

Filter matrix: \( M \times C K^2 = 16 \times 150 \)

Unrolled input matrix: \( C K^2 \times H_{out} W_{out} = 150 \times 100 \)

Input replication: \( 150 \times 100 / (6 \times 14^2) = 12.8 \times \)
LeNet-5 C5 Layer Unrolling Replication: 1×

LeNet-5 C5

- 5×5 input, C=16 features
- 5×5 filters (K=5)
- 1×1 outputs, M=120 features

Filter matrix: \( M \times CK^2 = 120 \times 400 \)

Unrolled input matrix: \( CK^2 \times H_{out} W_{out} = 400 \times 1 \)

Input replication: \( 400 \times 1 / (16 \times 5^2) = 1 \times \)
Reuse in 2D Tiled Convolution

Consider 2D convolution with shared memory tiling.

- Each output tile has $TILE\_WIDTH^2$ elements.
- Each input tile has $(TILE\_WIDTH + K - 1)^2$ elements.
- Number of input element accesses is $TILE\_WIDTH^2 \times K^2$.
- Input value reuse is then $K^2 \times TILE\_WIDTH^2 / (TILE\_WIDTH + K - 1)^2$. 

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Convolution Reuse Matches Unrolling Replication!

Theoretical reuse of input values for LeNet-5 is then
- C1: $19.1 \times$ (32×32 input tiles)
- C2: $12.8 \times$ (14×14 input tiles)
- C3: $1 \times$ (5×5 input tiles)

**Remember those numbers?**

Those are the replication factors for the unrolled inputs. If one unrolls conceptually, one obtains most/all of this reuse as well. If one actually replicates, this reuse is lost.
What about input reuse with matrix multiplication?

Filter matrix is $M \times CK^2$,  
- so **theoretical reuse is M**  
- for each value from the input feature map matrix  
- (also limited by output tile size).

In LeNet-5, C1 gives 6, C3 gives 16, and C5 gives 120,  
- which are the **numbers M of output features**.  
- Note that C5 is actually a matrix-vector multiply.
Conceptual Unrolling May Help Performance

So while **actually unrolling the inputs**
• **may** provide a **boost** in **coalesced accesses**, 
• the **cost** of doing so **is high**, both
  – in terms of preprocessing overhead
  – as well as loss of reuse.

However, performing **conceptual unrolling**
• during matrix tile load
• to transform convolution into matrix multiply
• often **produces** a **better** end **result**, 
• particularly **when coupled with joint tiling**.

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Specialization for the LeNet-5 C1 Layer

Let’s see how we might write **specialized code** for the **LeNet-5 C1** layer. C1 looks like this…

![Diagram showing the structure of the LeNet-5 C1 layer with unrolled inputs, weights, and shared memory tiling.](image)
Block Organization for LeNet-5 C1 Layer

How many threads in a block?
Maybe $4 \times 28 = 112$?

So a **block produces four rows** of each output feature (coarsened across all output features).

And we need **7 thread blocks**.
To do so, the **block needs 8 rows of input.**
That’s $8 \times 32 = 256$ elements.

For simplicity, let’s use **one tile** and **128 threads** doing **two loads**, with only **112 threads computing** output.
Pseudo-Code for LeNet-5 C1 Layer

Initialize 6 output element sums
Load to shared memory
Load to shared memory
Barrier
If computing (tx < 112)
  loop over 25 mask positions
    loop over 6 output features
      read a weight
      multiply-and-add to an output
    loop over 6 output features
  store element
Coarsening Does Not Match Tiling Choices

Oops!

We coarsened the wrong way!

Every thread loads every weight!

But those are in constant memory, and

• used in exactly the same order
• by every thread
• in every block—should be ok.
Mini-Batching May be Needed to Fill GPU

Mini-batching increases work done by each kernel launch:
• group $B$ independent inputs, then
• compose $B \times C$ input features into a larger input matrix, and
• solve in parallel.

Earlier example used Z dimension for mini-batch.
Some Other Optimizations

• Use streams to overlap
  – reading next set of input feature maps,
  – with processing the current set,
  – and writing the previous set of outputs.

• Create unrolled matrix elements only when they are loaded into shared memory.

• For large inputs, use algorithms such as FFT to implement convolution.
Rules for Lab 7 Performance Boosting

• In Lab 7, you **MAY optimize** for Titan V **based on** the **test input parameters** given (including the number of images)—one can imagine doing so with C++ templates, although it sounds a little painful.

• You **MAY NOT optimize for all weights being equal to 1**; your code **MUST** use stored weights.

• You **PROBABLY want to copy** the **weights to constant memory** for your kernel.
Training, Error and Loss Function

- Now it’s time to look at training.
- Where do the weights come from?
- A neural network is basically a fancy version of curve fitting. Try to pick the weights (coefficients) to best approximate a desired (but discrete and sparse) set of outcomes.
- Much easier to optimize if the functions are continuous, differentiable, and have non-zero derivatives.
- The network defines a function (remember that we already designed the network to provide continuous, differentiable functions with non-zero derivatives), but we have to decide how to measure the difference between a desired outcome and an actual outcome.
Training, Error and Loss Function

• Typically, we do so by casting an “answer” into a vector space, then using a simple measure—sum of absolute differences, sum of squared differences, or something similar—to quantify error.
• Recall that the training data set is labeled with the “correct answers.”
  – In Lenet-5, labels give the correct digit in the image. The label information can be used to generate the “correct” output of the last stage: the “correct” probability values of the 10-element vector.

• Then we can iteratively refine weights until we reach an acceptable solution—that’s training.
Choosing an Error Function

Many error functions are possible. For example, \textbf{given label} $T$ (digit $T$),

- $E = 1 - k[T]$,
- the \textbf{probability of not classifying as} $t$.

\textbf{Alternatively}, since our categories are numeric, we can \textbf{penalize quadratically}:

\[
E = \sum_{j=0}^{c-1} k[j](j - T)^2
\]

Let’s \textbf{go with the latter}.
Training, Error and Loss Function

• It’s worth mentioning that the training error and the intuitive success rate of a classifier may not be the same, but they need to be close enough that training based on the error function improves the success rate.

• In particular, given a set of test images and a classifier, the intuitive definition of success is that the classifier’s answer matches the (unique, labeled) human answer, and we might report the percentage of successes based on that measure. However, that’s a hard max function (what’s the discrete category with the highest probability? Also known as argmax), which is piecewise differentiable but has zero derivative wherever a derivative is defined, and thus cannot easily be used for training.
Training, Error and Loss Function

- For each training image, the final stage of the network calculates the loss function (error).

- Given a sequence of training images, we can calculate the loss function of the sequence as an average of the errors for individual images.

- If the loss function value is too high, we start back-propagation and adjust all the weights used in all the layers.
Forward and Backward Propagation

Forward:
\[ f_1 = W_1 \cdot x + b_1 \]
\[ y = \text{sigmoid}(f_1) \]
\[ f_2 = W_2 \cdot y + b_2 \]
\[ z = \text{sigmoid}(f_2) \]
\[ k = \text{softmax}(z) \]

Backward:
\[ \frac{dE}{df_2} \]
\[ \frac{dE}{dy} \]
\[ \frac{dE}{df_1} \]
\[ \frac{dE}{dz} \]
\[ \frac{dE}{dk} \]
Stochastic Gradient Descent

How do we calculate the weights?

One common answer: stochastic gradient descent.

1. Calculate
   – derivative of sum of error $E$
   – over all training inputs
   – for all network parameters $\Theta$.

2. Change $\Theta$ slightly in the opposite direction (to decrease error).

3. Repeat.
Stochastic Gradient Descent

More precisely,

1. For every input \( X \),
2. evaluate network to compute \( k[i] \) (forward),
3. then use \( k[i] \) and label \( T \) (target digit) to compute error \( E \).
4. Backpropagate error derivative to find derivatives for each
Parameter Updates and Propagation

\[ f_{c1} = W_1 \cdot x + b_1 \]

\[ \frac{dE}{df_{c1}} \quad \frac{dE}{dy} \quad \frac{dE}{df_{c2}} \quad \frac{dE}{dy} \quad \frac{dE}{dz} \quad \frac{dE}{df_{c2}} \quad \frac{dE}{dz} \quad \frac{dE}{dk} \quad \frac{dE}{dk} \]

Need propagated error gradient (from backward pass)

\[ \frac{dE}{dW_1} = \frac{dE}{df_{c1}} \cdot \frac{dW_1}{df_{c1}} = \frac{dE}{df_{c1}} \cdot x \]

Need input (from forward pass)
Error and Gradient of Error

- Error (loss function) can be measured in many ways
  - E.g., \( E = (1/n) \sum_{i} (|Y_i - \hat{Y}_i|) \)
  - Average of L1 distance for a sequence of output images \( Y_1, Y_2, \ldots, Y_i \)

- Gradient of error is the rate at which the error changes with respect to the variables – inputs, outputs, weights, intermediates.
  - The gradient of error with respect to a variable is the partial derivative of the loss function with respect to that variable
  - The gradient of error with respect to the output \( Y_i \) in our example is \( \partial E/\partial Y_i = \text{simply } 1/n \)

- Intuitively, it gives the rate at which the error changes when the value of the output vector changes – \( \partial E/\partial Y_i \)
Overview of Gradient Back-Propagation

• Back-propagation is done for all images in the sequence for which the loss function value was generated.

• We put each input image through the forward path. This generates input feature maps $X_i$ and output feature maps $Y_i$ for all layers.
  – We will just use $X$ and $Y$ for simplicity for the rest of this lecture.

• The process propagates the gradient of the error with respect to the output from the last layer ($\partial E/\partial Y$) towards the first layer through the network.

• Each layer starts with $\partial E/\partial Y$ – gradient with respect to its output feature maps and computes:
  – $\partial E/\partial W$ – gradient of error with respect to its weights
  – $\partial E/\partial X$ – gradient of error with respect to its input feature maps, this becomes the $\partial E/\partial Y$ of the previous layer.
Back Propagation using Chain Rule

• To understand how the error changes with respect to the weights (\( \partial E / \partial W \)) or the inputs (\( \partial E / \partial X \)) in each layer, we can apply the Chain Rule in calculus:

\[ z = f(y) \text{ and } y = g(x), \text{ then} \]

\[ \frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} = f'(y)g'(x) = f'(g(x))g'(x) \]
Back Propagation using Chain Rule (Example)

- \( G = W \times X + B \) //Network
- \( Y = S(G) \) //Final activation layer
- \( E = \text{sum} | Y - \hat{Y} | \) //Loss function
- \( \hat{Y} \) is the true label and \( S \) is the Sigmoid function

- \( \frac{\partial E}{\partial W} : \)

\[
\frac{\partial E}{\partial W} = \frac{\partial E}{\partial Y} \times \frac{\partial Y}{\partial S} \times \frac{\partial S}{\partial G} \times \frac{\partial G}{\partial W}
\]

- \( \frac{\partial E}{\partial X} : \)

\[
\frac{\partial E}{\partial X} = 1 \times 1 \times S' \times X
\]

\[
\frac{\partial E}{\partial X} = \frac{\partial E}{\partial Y} \times \frac{\partial Y}{\partial S} \times \frac{\partial S}{\partial G} \times \frac{\partial G}{\partial X}
\]

\[
= 1 \times 1 \times S' \times W
\]
Backpropagation

Remember that $Y$ is a linear sum of $X$ values over channels (for each output feature). Derivatives are $W$ values.

$Y = W \cdot X$

\[
\frac{dE}{dX} = \frac{dE}{dY} \frac{dY}{dX} = W \cdot \frac{dE}{dY}
\]
Calculating dE/dX from dE/dY

```c
void convLayer_backward_dgrad(int B, int M, int C, int H, int W, int K, float *dE_dY, float *W, float *dE_dX) {
    int H_out = H - K + 1;             // calculate H_out, W_out
    int W_out = W - K + 1;
    for (int b = 0; b < B; ++b) {      // for each image
        for (int c = 0;  c < C; ++c)     // for each input channel
            for (int h = 0; h < H; ++h)    // for each input pixel (two loops)
                for (int w = 0; w < W; ++w)
                    dE_dX[b, c, h, w] = 0.0f;  // initialize to 0
        for (int m = 0;  m < M;  ++m)          // for each output feature map
            for (int h = 0; h < H_out; ++h)      // for each output value (two loops)
                for (int w = 0; w < W_out; ++w)
                    for (int c = 0;  c < C; ++c)     // for each input channel
                        for (int p = 0; p < K; ++p)      // for each element of KxK filter (two loops)
                            for (int q = 0; q < K; ++q)
                                dE_dX[b, c, h + p, w + q] += dE_dY[b, m, h, w] * W[m, c, p, q];
    }
}
```
Backpropagation

\[ Y = W \cdot X \]

\[
\frac{dE}{dW} = \frac{dE}{dY} \frac{dY}{dW} = \frac{dE}{dY} \cdot X
\]

Remember that \( Y \) is a linear sum of \( X \) values over channels (for each output feature). Derivatives are \( W \) values.
Calculating \( \frac{dE}{dW} \)

```c
void convLayer_backward_wgrad(int B, int M, int C, int H, int W, int K, float *dE_dY, float *X, float *dE_dW) {
    const int H_out = H - K + 1;             // calculate H_out, W_out
    const int W_out = W - K + 1;
    for (int b = 0; b < B; ++b) {            // for each image
        for(int m = 0; m < M; ++m)             // for each output feature map
            for(int c = 0; c < C; ++c)            // for each channel
                for(int p = 0; p < K; ++p)         // for each element of KxK filter (two loops)
                    for(int q = 0; q < K; ++q)
                        dE_dW[b, m, c, p, q] = 0.0f;   // initialize to 0
            for(int m = 0;  m < M;  ++m) // for each output feature map
                for(int h = 0; h < H_out; ++h) // for each output value (two loops)
                    for(int w = 0; w < W_out; ++w)
                        for(int c = 0; c < C; ++c)            // for each channel
                            for(int p = 0; p < K; ++p) // for each element of KxK filter (two loops)
                                for(int q = 0; q < K; ++q)
                                    dE_dW[b, m, c, p, q] += X[b, c, h + p, w + q] * dE_dY[b, m, h, w];
    }
}
```
What is $\partial E/\partial W$ for a conv layer?

- It is a matrix that gives the influence of each $W$ element on the error.
  - It is the same dimension as the $W$ matrix
- The calculation is about generating each element of the $\partial E/\partial W$ matrix
- For any given $p$ and $q$ values, $\partial E/\partial W[p,q]$ is the sum of $\partial E/\partial Y[h,w] * \partial Y[h,w]/\partial W[p,q]$ for all relevant $h$ and $w$ values
Back-propagation for a convolutional layer

- Convolutional network:
  \[ Y(h, w) = \sum_{p=1}^{P_{out}} \sum_{q=1}^{Q_{out}} (X(c, h + p, w + q) \times W(c, p, q)) \]

- \( \frac{\partial E}{\partial W} : \)
  \[
  \frac{\partial E}{\partial W} = \frac{\partial E}{\partial Y} \times \frac{\partial Y}{\partial W}
  \]
  \[
  \frac{\partial E}{\partial W}(c, p, q) = \sum_{h=1}^{H_{out}} \sum_{w=1}^{W_{out}} (\frac{\partial E}{\partial Y}(h, w) \times X(c, h + p, w + q))
  \]

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A Small Convolution Layer Example Generating $Y[0,0,0]$, $c=0$ (review)
Example

• In our example, \( W[0, 0, 0, 0] \) contribute to output
  – \( Y[0,0,0] \) with \( X[0,0,0] \) * \( W[0,0,0,0] \)
  – \( Y[0,0,1] \) with \( X[0,0,1] \) * \( W[0,0,0,0] \)
  – \( Y[0,1,0] \) with \( X[0,1,0] \) * \( W[0,0,0,0] \)
  – \( Y[0,1,1] \) with \( X[0,1,1] \) * \( W[0,0,0,0] \)
  – In general, the total number of terms is \( H_{out} \times W_{out} \)

• So, \( \partial E/\partial W[0,0,0,0] \) can be calculated as the sum of
  – \( \partial E/\partial Y[0,0,0] \) * \( \partial Y[0,0,0]/\partial W[0,0,0,0] = \partial E/\partial Y[0,0,0] \) * \( X[0,0,0] \)
  – \( \partial E/\partial Y[0,0,1] \) * \( \partial Y[0,0,1]/\partial W[0,0,0,0] = \partial E/\partial Y[0,0,1] \) * \( X[0,0,1] \)
  – \( \partial E/\partial Y[0,1,0] \) * \( \partial Y[0,1,0]/\partial W[0,0,0,0] = \partial E/\partial Y[0,1,0] \) * \( X[0,1,0] \)
  – \( \partial E/\partial Y[0,1,1] \) * \( \partial Y[0,1,1]/\partial W[0,0,0,0] = \partial E/\partial Y[0,1,1] \) * \( X[0,1,1] \)
Back-propagation for a convolutional layer

- Convolutional layer for a single image

\[ Y(h, w) = \sum_{p=1}^{P_{out}} \sum_{q=1}^{Q_{out}} (X(c, h + p, w + q) \times W^{m}(c, p, q)) \]

- Since each \( W(c, m, p, q) \) affects all elements of output \( Y(m, h, w) \), we should accumulate gradients over all pixels in the corresponding output feature

\[ \frac{\partial E^{m}}{\partial W}(c, p, q) = \sum_{h=1}^{H_{out}} \sum_{w=1}^{W_{out}} \left( \frac{\partial E^{m}}{\partial Y}(h, w) \times X(c, h + p, w + q) \right) \]
Calculating $\partial E/\partial W$

```c
void convLayer_backward_wgrad(int M, int C, int H, int W, int K,
    float* dE_dY, float* X, float* dE_dW)
{
    int m, c, h, w, p, q;
    int H_out = H - K + 1;
    int W_out = W - K + 1;
    for(m = 0; m < M; m++)
        for(c = 0; c < C; c++)
            for(p = 0; p < K; p++)
                for(q = 0; q < K; q++)
                    dE_dW[m, c, p, q] = 0;

    for(m = 0; m < M; m++)
        for(h = 0; h < H_out; h++)
            for(w = 0; w < W_out; w++)
                for(c = 0; c < C; c++)
                    for(p = 0; p < K; p++)
                        for(q = 0; q < K; q++)
                            dE_dW[m, c, p, q] += X[c, h + p, w + q] * dE_dY[m, h, w];
}
```

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A Small Convolution Layer Example Generating $\frac{\partial E}{\partial W}[0,0,_,_]$

$$\begin{array}{c|ccc}
1 & 2 & 0 & 1 \\
1 & 1 & 3 & 2 \\
0 & 2 & 2 & 0 \\
2 & 1 & 0 & 3 \\
\end{array}$$

$$\begin{array}{c|ccc}
0 & 2 & 1 & 0 \\
0 & 3 & 2 & 1 \\
1 & 1 & 0 & 2 \\
2 & 1 & 0 & 3 \\
\end{array}$$

$$\begin{array}{c|ccc}
1 & 2 & 1 & 0 \\
0 & 1 & 3 & 2 \\
3 & 3 & 2 & 0 \\
1 & 3 & 2 & 0 \\
\end{array}$$
A Small Convolution Layer Example Generating $\frac{dE}{dW}[0,0-2,_,_]$

\[
\begin{array}{cccc}
1 & 2 & 0 & 1 \\
1 & 1 & 3 & 2 \\
0 & 2 & 2 & 0 \\
2 & 1 & 0 & 3 \\
\end{array}
\]

\[
\begin{array}{cccc}
1 & 1 & 3 & 2 \\
0 & 2 & 2 & 0 \\
2 & 1 & 0 & 3 \\
\end{array}
\]

\[
\begin{array}{cccc}
5 & 6 & 5 \\
3 & 5 & 5 \\
3 & 3 & 5 \\
\end{array}
\]

\[
\begin{array}{cccc}
5 & 6 & 5 \\
3 & 5 & 5 \\
3 & 3 & 5 \\
\end{array}
\]

\[
\begin{array}{cccc}
5 & 6 & 5 \\
3 & 5 & 5 \\
3 & 3 & 5 \\
\end{array}
\]

\[
\begin{array}{cccc}
6 & ? & ? \\
? & ? & ? \\
? & ? & ? \\
\end{array}
\]

\[
\begin{array}{cccc}
6 & ? & ? \\
? & ? & ? \\
? & ? & ? \\
\end{array}
\]

\[
0 \cdot 1 + 2 \cdot 1 + 0 \cdot 1 + 3 \cdot 1 = 6
\]
A Small Convolution Layer Example Generating $\frac{dE}{dW}[0,0-2,\_,\_]$

\[
\begin{array}{cccc}
1 & 2 & 0 & 1 \\
1 & 1 & 3 & 2 \\
0 & 2 & 2 & 0 \\
2 & 1 & 0 & 3 \\
\end{array}
\]

\[
\begin{array}{cccc}
0 & 2 & 1 & 0 \\
0 & 3 & 2 & 1 \\
1 & 1 & 0 & 2 \\
2 & 1 & 0 & 3 \\
\end{array}
\]

\[
\begin{array}{cccc}
5 & 6 & 5 & \_ \\
3 & 5 & 5 & \_ \\
3 & 3 & 5 & \_ \\
\end{array}
\]

\[
\begin{array}{cccc}
6 & 8 & \? & \_ \\
\? & \? & \? & \_ \\
\? & \? & \? & \_ \\
\end{array}
\]

\[
2 \times 1 + 1 \times 1 + 3 \times 1 + 2 \times 1 = 8
\]

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A Small Convolution Layer Example Generating $dE/dW[0,0-2,_,_]$

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$dE/dW[0,0,_,_ ]$

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$dE/dY[0,_,_ ]$

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We can picture this computation in a more familiar way

$1*1 + 0*1 + 2*1 + 1*1 = 4$
A Small Convolution Layer Example Generating $\frac{dE}{dW}[0,0-2,__,_]$

<table>
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<th>$x[1,__,_]$</th>
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<th>$dE/dY[0,__,_]$</th>
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<td>2 1 0 3</td>
<td>3 3 5</td>
<td>? ?</td>
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</tbody>
</table>

$1 \times 1 + 0 \times 1 + 2 \times 1 + 1 \times 1 = 4$
A Small Convolution Layer Example Generating dE/dW[0,0-2,_,_]
A Small Convolution Layer Example Generating $\frac{dE}{dW}[0,0-2,\_,\_]$
A Single Output Feature and a Single Input Channel Are Needed
Matrix-vector multiplication can be used to compute each dE/dW channel per output.
A Small Convolution Layer Example Generating $\frac{dE}{dW}[0,0-2,._._]$
Adjusting Weights

- After the $\frac{dE}{dW}$ values at all feature map element positions are computed, weights are updated:

- For each weight value
  - $w(t+1) = w(t) - \lambda \left( \frac{dE}{dw} \right)$

- where $\lambda$ is the learning rate (hyper-parameter).
### Updating the Weights

$$W(t)$$

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$$W(t+1)$$

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</table>

$$- 0.01 \times \frac{dE}{dW[0,0,_,_]}$$

$$= \begin{bmatrix} 0.97 & 0.9 & 0.99 \\ 1.98 & 1.9 & 2.95 \end{bmatrix}$$

$$W[0,0,_,_]$$

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</table>

$$- 0.01 \times \frac{dE}{dW[0,1,_,_]}$$

$$= \begin{bmatrix} 0.98 & 1.9 & 2.99 \\ 0.97 & 0.9 & 0 \end{bmatrix}$$

$$W[0,1,_,_]$$

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$$- 0.01 \times \frac{dE}{dW[0,2,_,_]}$$

$$= \begin{bmatrix} 0.99 & 0.9 & 0.97 \\ 0.99 & 0 & 1.95 \end{bmatrix}$$

$$W[0,2,_,_]$$

$$\lambda = 0.01$$
Back-propagation $dE/dX$ for a convolutional layer

- Convolution Layer:
  - Multiple input features affect a single output feature
  - Each input feature affects multiple output features

- $dE/dX$:
  - Multiple output features propagate to a single input feature
  - Each one through a different $W$
Back-propagation $\frac{dE}{dX}$ for a convolutional layer

\[
\frac{\partial E}{\partial X}(c, h, w) = \sum_{m=1}^{M} \sum_{p=1}^{K} \sum_{q=1}^{K} (W(p, q) \ast \frac{\partial E}{\partial Y}(h - p, w - q))
\]

Now we need to iterate through all output features.
Example

- In our example, X[0, 1, 1] contribute to output
  - Y[0,1,1] with X[0,1,1] * W[0,0,0,0]
  - Y[0,1,0] with X[0,1,1] * W[0,0,0,1]
  - Y[0,0,1] with X[0,1,1] * W[0,0,1,0]
  - Y[0,0,0] with X[0,1,1] * W[0,0,1,1]
- In general, the total number of terms is K×K
- So, ∂E/∂X[0,1,1] can be calculated as the sum of
  - ∂E/∂Y[0,1,1] * ∂Y[0,1,1]/∂X[0,1,1]= ∂E/∂Y[0,1,1] * W[0,0,0,0]
  - ∂E/∂Y[0,1,0] * ∂Y[0,1,0]/∂X[0,1,1]= ∂E/∂Y[0,1,0] * W[0,0,0,1]
  - ∂E/∂Y[0,0,1] * ∂Y[0,0,1]/∂X[0,1,1]= ∂E/∂Y[0,0,1] * W[0,0,1,0]
  - ∂E/∂Y[0,0,0] * ∂Y[0,0,0]/∂X[0,1,1]= ∂E/∂Y[0,0,0] * W[0,0,1,1]
Calculating $\frac{dE}{dX}$

```c
void convLayer_backward_dgrad(int M, int C, int H, int W, int K,
    float* dE_dY, float* W, float* dE_dX)
{
    int m, c, h, w, p, q;
    int H_out = H – K + 1;
    int W_out = W – K + 1;
    for(c = 0;  c < C; c++)
        for(h = 0; h < H; h++)
            for(w = 0; w < W; w++)
                dE_dX[c, h, w] = 0.;
    for(m = 0;  m < M;  m++)
        for(h = 0; h < H_out; h++)
            for(w = 0; w < W_out; w++)
                for(c = 0;  c < C; c++)
                    for(p = 0; p < K; p++)
                        for(q = 0; q < K; q++)
                            dE_dX[c, h + p, w + q] += W[m, c, p, q] * dE_dY[m, h, w];
}
```
A Small Convolution Layer Example Generating $\frac{dE}{dX}[0,\_\_\_]$
A Small Convolution Layer Example Generating $dE/dX[0,_,_]$
A Small Convolution Layer Example Generating $\frac{dE}{dX}[0,_,_]$

$$\begin{array}{c}
\text{dE/dY}[0,_,_] \\
\text{dE/dY}[1,_,_] \\
\text{dE/dY}[2,_,_] \\
\end{array}$$

$$\begin{array}{c}
\text{W}[0,0,_,_] \\
\text{W}[0,1,_,_] \\
\text{W}[0,2,_,_] \\
\end{array}$$

$$\begin{array}{c}
\text{dE/dX}[0,_,_] \\
\text{dE/dX}[1,_,_] \\
\text{dE/dX}[2,_,_] \\
\end{array}$$

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A Small Convolution Layer Example Generating $dE/dX[0,_,_]$

- Let’s go back to our previous example that has only two $dE/dY$ contributions
A Small Convolution Layer Example Generating $dE/dX[0,_,_]$

- We can design this computation as convolution.
- But first we need to ready both $dE/dY$ and $W$ which are the inputs to $dE/dX$. 

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A Small Convolution Layer Example Generating $dE/dX[0,_,_]$
ANY QUESTIONS?