Project Idea for ECE 408: GPU Accelerated Ensemble Kalman Filter

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April 11, 2019
Space Weather: Solar Driven Variation of the Space Environment

- My research: combine data and physics models to characterize regions governing / affected by space weather
“The Day the Sun Brought Darkness”

- March 13, 1989: 9 hour power outage in the Québec region
- 6 million people affected
- Cause: geomagnetically induced currents (GICs) triggered by a severe geomagnetic storm (coronal mass ejection)
Space Exploration Depends on Ionospheric Modeling

- Ionospheric delay calibration $\rightarrow$ tracking precision $\rightarrow$ successful orbital insertion $\rightarrow$ Mars Science Laboratory (Curiosity) triumph
Space Weather Data Volume Continues to Expand and Diversify

- **Multibillion dollar** spacecraft fleet dedicated to space science
- Solar Dynamic Observatory downlinks **1.5 TB** daily
Signal Processing Advancements Required for Space Weather Specification

- **Objective:** space weather nowcast — spatial / temporal physical parameter specification

- Available resources:
  - Exploding (but *incomplete*) data volume
  - First-principles (but *idealized*) physics models / simulations

**Data assimilative model** = systematic combination of data and prior physical constraints
Data Assimilation is the Foundation of Numerical Weather Prediction

Interconnected factors driving quantitative improvements:

- Increasing data volume
- Available computational resources
- **Statistical methodology** advancements (assimilative modeling)
Nowcasting of the Inner Solar Atmosphere

Data \((4096^2 \text{ images, } 1.5 \text{ TB daily})\)

Voxel grid
\((100 \times 100 \times 100 = 1 \text{ million parameters})\)

State nowcast — time snapshot of solar atmosphere density / temperature
Tomography: Volumetric Imaging From Line-Of-Site Integrated Measurements

- Computed tomography = fixed patient, rotating X-ray source, scan time typically minutes
- Solar tomography = rotating patient (Sun), fixed sensor (satellite), scan time of 2 weeks (13° rotation per day)
Linear State-Space Model: Framework for Heliospheric Data Assimilation

Forecast model:
\[ x_{i+1} = F_i x_i + u_i \]

Measurement model:
\[ y_i = H_i x_i + v_i \]

Data \( y_i \)

System state \( x_i \)

GOAL:
Nowcast the system state (what is \( x_i \)?)
(Nowcast = the system state now given the data until now)
Kalman Filter: Classic Linear State Estimation Procedure

- The Kalman filter (1960) is the optimal solution method

- Kalman filter is the recursive algorithm to compute
  - System state nowcast $\hat{x}_{i|i}$, an $N$ vector
  - Estimate error covariance $P_{i|i}$, an $N \times N$ matrix,
  - Error-bars are the diagonal elements of $P_{i|i}$
Ensemble Kalman Filter is a Monte Carlo Kalman Filter

- For 1 million state parameters ($N$), the KF requires almost 2 TB
- The ensemble KF is a Monte Carlo approximation to the KF

\[
\text{Memory} \approx \frac{N^2}{2}
\]

\[
\text{Memory} = NL
\]

\[
L = \infty
\]

\[
L = 30
\]

\[
L = 10
\]

- Larger ensemble ($L$) yields a closer approximation to the Kalman filter, but requires more memory and computation
Ensemble Kalman Filter Algorithm

Corrector \( \tilde{x}_{i|i} \rightarrow \tilde{x}_{i|i} \)

\[
\tilde{x}_{i|i-1} = \frac{1}{L} \sum_{l=1}^{L} \tilde{x}_{i|i-1}^l, \quad z_{i|i-1}^l = \tilde{x}_{i|i-1}^l - \tilde{x}_{i|i-1}
\]

\[
B_i \triangleq (C_i \circ \tilde{P}_{i|i-1}) H_i^T = \left[ C_i \circ \frac{1}{L-1} \sum_{l=1}^{L} z_{i|i-1}^l(z_{i|i-1}^l)^T \right] H_i^T
\]

\[
\tilde{K}_i = B_i (H_i B_i + R_i)^{-1}
\]

\[
x_{i|i} = x_{i|i-1} + \tilde{K}_i (y_i - H_i x_{i|i-1}), \quad y_i \text{i.i.d.} \sim N(y_i, R_i)
\]

Forecaster \( \tilde{x}_{i|i} \rightarrow \tilde{x}_{i+1|i} \)

\[
x_{i+1|i} = F_i x_{i|i} + u_i
\]

\[
u_i \text{i.i.d.} \sim N(0, Q_i)
\]

- Error covariance is never **explicitly** computed (need \( B_i \) instead)
- Ensemble size \( L \) controls computation trade-off
- See Butala et al., IEEE TIP, 2009 for proof of convergence
Critical Computational Step

\((AA^T) \circ (ZZ^T)\)

- The \(\circ\) operator is element-by-element matrix multiply

- The matrix \(Z \in \mathbb{R}^{N \times L}\) is dense and “tall but narrow,” i.e., \(N \gg L\) \((N = 10^6 \text{ and } L = 10^1 \text{ or } 10^2)\)

- The matrix \(A\) is sparse and encodes a 3-D convolution (the matrix has block Toeplitz structure)
Lemma 4.17: Let $A$ be an $M \times N$ matrix and $B$ be an $M \times P$ matrix. Then

$$ (A A^T) \odot (B B^T) = (A^T \odot B^T)^T (A^T \odot B^T) \quad (4.103) $$

where $\odot$ denotes the Khatri-Rao matrix product defined in Appendix C.

Definition C.1: Let $A$ be a $P \times N$ matrix and $B$ be a $M \times N$ matrix. The Khatri-Rao product of the matrices $A$ and $B$ is defined by

$$ A \odot B = \begin{bmatrix} [A](:,1) \otimes [B](:,1) & \cdots & [A](:,N) \otimes [B](:,N) \end{bmatrix} \quad (C.1) $$

where $\otimes$ is the Kronecker matrix product, $[\cdot](:,n)$ selects the $n$th column of the matrix argument, and $A \odot B$ has dimensions $PM \times N$. 
Ensemble Kalman Filter Applications

- Numerical weather prediction
- Soil moisture / agriculture / crop science
- Ecosystem and carbon cycle
- Oceanography / hydrology
- Hurricane / tornado modeling
- Disease transmission
- Medical imaging
- Radiation belt / thermosphere / ionosphere modeling
- ...

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