

Multi-Digit Addition is Correct by Induction

Probably a **proof by induction**...

- 1. You know how to add 1-digit numbers. Verifying an addition table suffices.
- 2. GIVEN that you can add N-digit numbers, show (based, for example, on place value) that you can add (N+1)-digit numbers.

But you didn't know about proof by induction $^{\circ}$ when you learned how to add,

 $^{\circ}$ so you've probably never seen a proof.

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slide 3

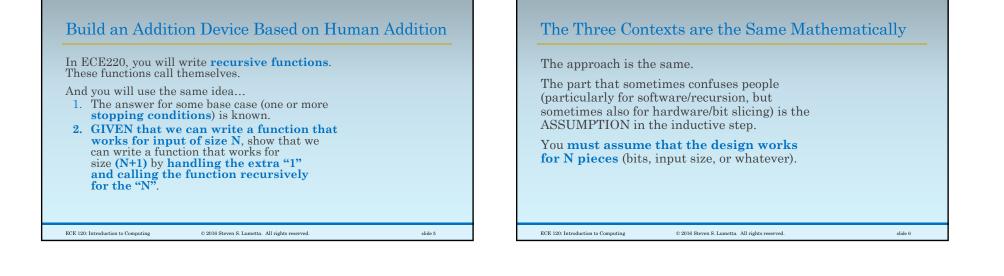
The Ripple Carry Adder is Also Correct by Induction

When we designed a ripple carry adder, we also **assumed proof by induction**.

- 1. We know how to add one bit. We made a truth table (a binary addition table).
- 2. GIVEN that we can build an N-bit adder, show that we can build an (N+1)-bit adder by attaching a full (1-bit) adder to an (N-bit) adder.

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slide 4



All Three Approaches Require a "Leap of Faith"

You don't need to design the system all at once for N (other than some base case).

In other words,

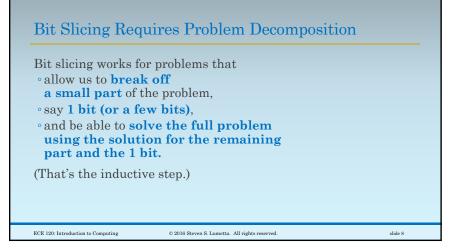
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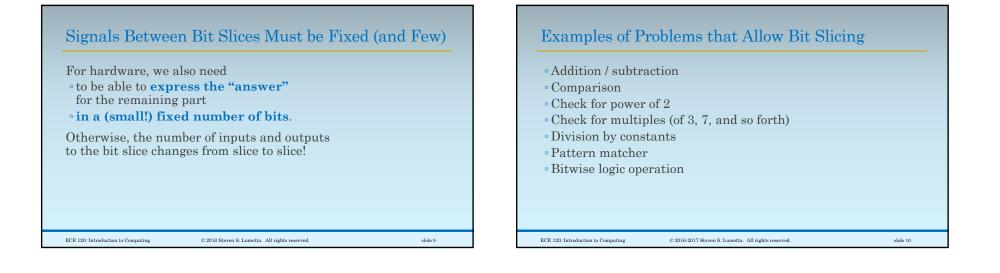
- you must make a "leap of faith" and
- assume that your answer works
- before you actually design it!

People sometimes have trouble making such an assumption, but it's just a **standard part of an inductive proof**.

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slide 7





When Can't We Used Bit Slicing?

One example: when the answer depends on ALL of the other bits (can't summarize an answer for N bits).

For example, can you create a bit-sliced prime number identifier?

 $A_{N-1} A_{N-2} \dots A_5 \leftarrow (summary) \ 0 \ 1 \ 0 \ 0 \ 1$

What information do you pass to bit 5?

All 5 bits? 01001? I have no idea!

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slide 11