

University of Illinois at Urbana-Champaign
Dept. of Electrical and Computer Engineering

ECE 120: Introduction to Computing

Boolean Expression Terminology

Let's Review and Define Some New Terms

literal a variable or its complement

examples: A , A' , B , B' , C , C'

sum several terms ORed together

examples: $A + B$, $AB + B(C + D) + A'C$,
 $A'B' + D(A \oplus B)(C + A')$

product several terms ANDed together

examples: AB , $(A + B)(B + CD)(A' + C)$,
 $(A' + B')(D + (A \oplus B) + CA')$

Minterms Were Useful for Proving Logical Completeness

minterm on N inputs

a product in which each variable or
its complement appears exactly
once (no other factors)

examples: AB' , $A'B$, AB (on inputs A , B)

$AB'C$, $AB'C'$, $A'BC'$

(on inputs A , B , C)

A Maxterm Produces a Function with One Zero Row

maxterm on N inputs

a sum in which each variable or
its complement appears exactly
once (no other terms)

examples: $(A + B')$, $(A' + B)$, $(A + B)$

(on inputs A , B)

$(A + B' + C)$, $(A + B' + C')$,

$(A + 'B + C')$

(on inputs A , B , C)

Sum-of-Products (SOP) Form is Quite Common

sum-of-products (SOP)

a sum (OR)
of products (AND)
of literals

examples: $AB + BC$,

$$AB' + C + A'C'D'$$

but NOT $A(B + C) + D$

Product-of-Sums (POS) Form is Also Common

product-of-sums (POS)

a product (AND)
of sums (OR)
of literals

examples: $(A + B)(B + C)$,

$$(A + B')C(A' + C' + D')$$

but NOT $(A + BC)D$

Canonical Forms Allow Easy Comparison, But Are Too Big

canonical SOP

a sum of minterms; the expression
produced by the logical
completeness construction

canonical POS

a sum of maxterms

What does canonical mean?

Unique (if we assume an ordering on variables).

Too many terms to be of practical value.

Do You Know Mathematical Implication?

What does $A \rightarrow B$ mean?

A implies B.

In other words: **if A is true, B is also true.**

What if A is false?

In that case, **is $A \rightarrow B$ true or false?**

If A is false, $A \rightarrow B$ is true.

So the Following Odd Statements are True

All **purple elephants** can fly.
 (X is a **purple elephant** \rightarrow X can fly.)

Students who score **above 125%**
 in ECE120 fail the class.
 (X scored **above 125%** \rightarrow X fails.)

In both, **the premise is false for any X**, so
 the **implications are true**.

One Function Can Imply Another

A function **G** is an **implicant of** a second
 function **F** iff **G** operates on the same
 variables as **F** and **G** \rightarrow **F**.

In other words, every row

- with an output of 1 in **G**'s truth table
- also has an output of 1 in **F**'s truth table.

0 rows in **G**'s truth table do not matter.

For Our Purposes, Implicants are Products of Literals

In digital design, we only refer to
 products of literals as implicants.

So we will **assume that an implicant
 can be written as a product of literals**.

We Can Use Implicants to Simplify Functions

As a first step towards simplifying
 a function **F**, we can ask:

**Given an implicant G of F, can we
 remove any of its literals and obtain
 another implicant of F?**

For example, take **F = AB'C + ABC' + ABC**.

The first term (**AB'C**) is an implicant.

Can we remove any literals?

Try to Remove Each Literal to Find Only AC Implies F

	A	B	C	F	B'C	AC	AB'
Start from $AB'C$ and try to remove each literal.	0	0	0	0	0	0	0
	0	0	1	0	1	0	0
B'C is not an implicant.	0	1	0	0	0	0	0
	0	1	1	0	0	0	0
AC is an implicant.	1	0	0	0	0	0	1
	1	0	1	1	1	1	1
AB' is not an implicant.	1	1	0	1	0	0	0
	1	1	1	1	0	1	0

We Remove as Many Literals as We Can

So we can simplify F by replacing $AB'C$ with AC :

$$F = AC + ABC' + ABC$$

Checking the second term (ABC'), we find that we can eliminate C' to obtain:

$$F = AC + AB + ABC$$

In the third term (ABC), we can eliminate B or C , but not both. Let's pick B .

$$F = AC + AB + AC$$

Prime Implicants Have a Minimal Number of Literals

$$F = AC + AB + AC$$

But now we have a duplicate term, which we can eliminate to arrive at a simple form for F :

$$F = AC + AB$$

We can remove no more literals.

One more definition: An implicant G of F is a **prime implicant of F** iff **none of the literals in G can be removed** to produce other implicants of F .

AB and AC are prime implicants of F .

To Simplify, Write Function as a Sum of Prime Implicants

The conclusion is obvious:

To simplify a function F , write it as a sum of prime implicants.

Enjoy the algebra.

Good luck!

(Next time, we'll develop a graphical tool that lets us skip the algebra.)